Mirror Images of String Cosmologies

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Abstract

A discrete symmetry of the four–dimensional string effective action is employed to derive spatially homogeneous and inhomogeneous string cosmologies from vacuum solutions of general relativity that admit two commuting spacelike Killing vectors. In particular, a tilted Bianchi type V cosmology is generated from a vacuum type VI_h solution and a plane wave solution with a bounded and oscillating dilaton field is found from a type VII_h model. Further applications are briefly discussed.

PACS NUMBERS: 98.80, 04.50.+h, 11.25.Mj

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The cosmological implications of string theory are currently receiving considerable attention [1, 2, 3, 4, 5]. Although a non–perturbative formulation of the theory has yet to be established, the low–energy effective action provides a framework for describing the evolution of the very early universe immediately below the string scale. This era represents a natural environment for establishing observational constraints on the theory and addressing central questions such as whether it admits realistic inflationary universes [1, 2].

The search for cosmological solutions to the string equations of motion is therefore well motivated. The majority of studies to date have investigated spatially homogeneous backgrounds, including the isotropic Friedmann–Robertson–Walker (FRW) cosmologies [3], the Bianchi models [4] and Kantowski–Sachs universes [5]. Since our understanding of the geometry of the universe near the string scale is incomplete, however, it is important to consider the role of inhomogeneities in string cosmology. The field equations for inhomogeneous backgrounds are in general very difficult to solve, but further progress can be made by considering models where homogeneity is broken along one spatial direction. String cosmologies satisfying this property were recently derived by employing a variety of methods [6].

The purpose of the present letter is to show that a wide class of homogeneous and inhomogeneous string cosmologies may be generated in a very straightforward manner from known vacuum solutions of general relativity. We employ a discrete Z_2 symmetry that arises in the Neveu–Schwarz/Neveu–Schwarz (NS–NS) sector of the effective action when the space–time admits two commuting spacelike Killing vectors [7].

To lowest order in the inverse string tension, α', the four–dimensional effective NS–NS action is [8]

$$S = \int d^4x\sqrt{-G}e^{-\Phi}\left[R(G) + (\nabla\Phi)^2 - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\right], \qquad (1)$$

where R is the Ricci curvature scalar of the space–time with metric $G_{\mu\nu}$, Φ is the dilaton field, $H_{\mu\nu\lambda} \equiv \partial_{[\mu}B_{\nu\lambda]}$ is the field strength of the antisymmetric two–form potential, $B_{\mu\nu}$, and $G \equiv \det G_{\mu\nu}$. It is convenient to perform the conformal transformation to the 'Einstein frame':

$$g_{\mu\nu} \equiv \Theta^2 G_{\mu\nu}, \qquad \Theta^2 \equiv e^{-\Phi}. \qquad (2)$$

The effective action (1) then takes the form

$$S = \int d^4x\sqrt{-g}\left[R(g) - \frac{1}{2}(\nabla\Phi)^2 - \frac{1}{12}e^{-2\Phi}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\right], \qquad (3)$$

where $g \equiv \det g_{\mu\nu}$.

In four–dimensions, the field strength of the two–form potential is dual to a one–form. We may therefore apply a duality transformation:

$$H^{\mu\nu\lambda} \equiv \epsilon^{\mu\nu\lambda\kappa}e^{2\Phi}\nabla_\kappa\sigma, \qquad (4)$$

where σ represents a pseudo–scalar axion field and $\epsilon^{\mu\nu\lambda\kappa}$ is the covariantly constant four–form. The action (3) is then equivalent to the non–linear sigma–model

$$S = \int d^4x\sqrt{-g}\left[R + \frac{1}{4}\mathrm{Tr}\left(\nabla N \nabla N^{-1}\right)\right], \tag{5}$$

where

$$N \equiv \begin{pmatrix} e^{\Phi} & \sigma e^{\Phi} \\ \sigma e^{\Phi} & e^{-\Phi} + \sigma^2 e^{\Phi} \end{pmatrix} \tag{6}$$

is a symmetric $\mathrm{SL}(2,R)$ matrix. The dilaton and axion parametrize the $\mathrm{SL}(2,R)/\mathrm{U}(1)$ coset and Eq. (5) is invariant under global $\mathrm{SL}(2,R)$ transformations:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}, \qquad \tilde{N} = \Omega N \Omega^T, \tag{7}$$

where

$$\Omega \equiv \begin{pmatrix} d & c \\ b & a \end{pmatrix}, \qquad ad - bc = 1 \tag{8}$$

is a constant $\mathrm{SL}(2,R)$ matrix.

We consider the class of metrics defined by the block diagonal line element

$$ds^2 = h_{\alpha\beta}(x^\epsilon)dx^\alpha dx^\beta + \gamma_{ab}(x^\epsilon)dx^a dx^b, \tag{9}$$

where $\{x^\alpha = t, x\}$ and $\{x^a = y, z\}$. The functions $h_{\alpha\beta}$ and γ_{ab} depend only on the variables t and x and spatial homogeneity is broken along the x–direction [9, 10]. Eq. (9) admits two commuting spacelike Killing vectors, $\partial/\partial x^a$, and there exists an abelian group, G_2, of isometries. We refer to this class of metrics as G_2 models.

If all massless excitations in the string action (5) are functions only of t and x, we may integrate over the transverse coordinates to derive an effective two–dimensional action:

$$S = \int d^2x\sqrt{-h}e^{-\gamma}\left[R_2 + \frac{1}{2}(\nabla\gamma)^2 + \frac{1}{4}\mathrm{Tr}\left(\nabla f \nabla f^{-1}\right) + \frac{1}{4}\mathrm{Tr}\left(\nabla N \nabla N^{-1}\right)\right], \tag{10}$$

where R_2 is the Ricci curvature of the $(1+1)$–dimensional manifold with metric $h_{\alpha\beta}$, $h \equiv \det h_{\alpha\beta}$ and $\gamma \equiv -(\ln\det\gamma_{ab})/2$. The symmetric 2×2 matrix, $f_{ab} \equiv e^{\gamma}\gamma_{ab}$, is an element of $\mathrm{SL}(2,R)$ and may be expressed in the form

$$f \equiv \begin{pmatrix} e^{V} & \omega e^{V} \\ \omega e^{V} & e^{-V} + \omega^2 e^{V} \end{pmatrix}. \tag{11}$$

Thus, the scalar functions V and ω parametrize a second $\mathrm{SL}(2,R)/\mathrm{U}(1)$ coset.

The field equations derived from Eq. (10) admit an infinite–dimensional symmetry that can be identified infinitesimally with the $\mathrm{O}(2,2)$ current algebra [7, 11, 12]. The well–known non–compact, global $\mathrm{O}(2,2)$ and $\mathrm{SL}(2,R)$ symmetries may be embedded in this larger symmetry. As discussed by Bakas [7], however, there exists a further Z_2

symmetry that corresponds to a discrete interchange of the field content of the two $\mathrm{SL}(2,R)/\mathrm{U}(1)$ cosets in Eq. (10):

$$\tilde{N}_{ab} = f_{ab}, \qquad \tilde{f}_{ab} = N_{ab}. \tag{12}$$

The two–metric, $h_{\alpha\beta}$, and determinant, γ, are invariant under this transformation.

Eq. (12) is not part of the $\mathrm{SL}(2,R)$ transformation (7) because the Einstein–frame metric, $g_{\mu\nu}$, does not transform as a singlet. Moreover, it is not part of the global $\mathrm{O}(2,2)$ symmetry [13]. This symmetry leaves the shifted dilaton field, $\phi \equiv \Phi + \gamma$, invariant and, in general, the dilaton is not a singlet under Eq. (12), whereas γ is. In effect, Eq. (12) interchanges the dilaton and axion fields with the components of the metric on the surfaces of orthogonality. The axion field is interchanged with the off–diagonal component, ω, in Eq. (11) and the dilaton field with the function V.

A string background is parametrized in terms of the massless degrees of freedom $\{G_{\mu\nu}, \Phi, B_{\mu\nu}\}$. In this sense, a vacuum solution to Einstein gravity may be viewed as the subset of string backgrounds, $\{G_{\mu\nu}, 0, 0\}$, where the dilaton and two–form potential are trivial. Application of Eq. (12) to a given string background then leads to a new solution $\{\tilde{G}_{\mu\nu}, \tilde{\Phi}, \tilde{B}_{\mu\nu}\}$ that admits a different space–time interpretation. The latter solution may be viewed as the 'mirror image' of the former.

We now employ the invariance (12) to derive inhomogeneous string cosmologies containing both dilaton and axion fields from vacuum G_2 solutions. The Gowdy models are the class of vacuum G_2 cosmologies where the spacelike hypersurfaces are compact [14]. The allowed topologies are a three–torus, $S^1 \times S^1 \times S^1$, a hypertorus, $S^1 \times S^2$, and a three–sphere, S^3. Without loss of generality, the toroidal models may be written in the form

$$ds^2 = e^k\left(-dt^2 + dx^2\right) + t\left(e^p dy^2 + e^{-p} dz^2\right), \tag{13}$$

where k and p represent the longitudinal and transverse parts of the gravitational field, respectively, and are functions of t and x. The vacuum Einstein field equations then reduce to

$$\ddot{p} + \frac{1}{t}\dot{p} - p'' = 0 \tag{14}$$

$$\dot{k} = -\frac{1}{2t} + \frac{t}{2}\left(\dot{p}^2 + p'^2\right) \tag{15}$$

$$k' = t\dot{p}p', \tag{16}$$

where a dot and prime denote $\partial/\partial t$ and $\partial/\partial x$, respectively. The general solution to Eqs. (14)–(16) consistent with the toroidal boundary conditions is known [14, 15].

Since the metric (13) is diagonal, a direct application of Eq. (12) would generate a string solution with only the dilaton field excited. The simplest method for introducing an off–diagonal term in the metric is to perform an $\mathrm{SL}(2,R)$ transformation in the two–space of the Killing vectors, $\partial/\partial x^a$:

$$\tilde{f} = \Theta f \Theta^T, \qquad \Theta \equiv \begin{pmatrix} D & C \\ B & A \end{pmatrix}, \tag{17}$$

where $AD - BC = 1$ and all other variables are invariant [16]. The transverse metric (11) then transforms to

$$e^{\tilde{V}} = C^2 e^{-V} + D^2 e^V \tag{18}$$

$$\tilde{\omega} = \frac{ACe^{-V} + BDe^V}{C^2 e^{-V} + D^2 e^V}. \tag{19}$$

Eq. (17) does not commute with the discrete transformation (12) and the two may be employed together to generate the axion field. Applying Eq. (17), followed by the discrete transformation (12), leads to a new class of inhomogeneous string cosmologies with non–trivial dilaton and axion fields:

$$ds^2 = e^k \left(-dt^2 + dx^2\right) + t\left(dy^2 + dz^2\right) \tag{20}$$

$$\Phi = \ln\left[C^2 e^{-p} + D^2 e^p\right] \tag{21}$$

$$\sigma = \frac{ACe^{-p} + BDe^p}{C^2 e^{-p} + D^2 e^p}, \tag{22}$$

where p solves Eq. (14). The metric in the string frame is then given by substituting Eqs. (20) and (21) into Eq. (2):

$$dS^2 = e^{k+\Phi}\left(-dt^2 + dx^2\right) + te^{\Phi}\left(dy^2 + dz^2\right). \tag{23}$$

There is no preferred direction in the transverse space in Eq. (20) and this implies the existence of a one–parameter isotropy group in addition to the G_2 isometry group. The mirror images of the toroidal Gowdy models are therefore locally rotationally symmetric (LRS) inhomogeneous cosmologies with plane symmetry. We remark that a similar analysis may be applied to the hypertorus and three–sphere Gowdy models.

We now discuss further applications of Eq. (12) within the context of the spatially homogeneous cosmologies that admit an abelian subgroup G_2 of isometries. This includes the Bianchi types I–VII and the LRS types VIII and IX [17]. The general spatially homogeneous vacuum solution with a simply transitive Lie group $G_3 = \Re^3$ is the type I Kasner solution [18]:

$$ds^2 = -dt^2 + t^{2p_1}dx^2 + t^{2p_2}dy^2 + t^{2p_3}dz^2, \tag{24}$$

where $\sum_{i=1}^3 p_i = \sum_{i=1}^3 p_i^2 = 1$. Applying Eqs. (17) and Eq. (12) yields the LRS type I solution:

$$ds^2 = -dt^2 + t^{2p_1}dx^2 + t^{p_2+p_3}\left(dy^2 + dz^2\right)$$

$$\Phi = \ln\left[C^2 t^{p_3-p_2} + D^2 t^{p_2-p_3}\right]$$

$$\sigma = \frac{ACt^{p_3-p_2} + BDt^{p_2-p_3}}{C^2 t^{p_3-p_2} + D^2 t^{p_2-p_3}} \tag{25}$$

and this reduces to the general, spatially flat FRW string cosmology when $p_2 = (1-\sqrt{3})/3$ and $p_3 = (1+\sqrt{3})/3$ [3].

One of the simplest examples of a spatially curved, anisotropic model is the Bianchi type V. A vacuum type V solution was found by Joseph [19]:

$$ds^2 = \sinh 2t \left(-dt^2 + dx^2 + e^{-2x} \left[(\mathrm{tanh}t)^{\sqrt{3}} dy^2 + (\mathrm{tanh}t)^{-\sqrt{3}} dz^2 \right] \right) \tag{26}$$

and Eqs. (17) and (12) yield its mirror image:

$$\begin{aligned} ds^2 &= \sinh 2t \left[-dt^2 + dx^2 + e^{-2x} \left(dy^2 + dz^2 \right) \right] \\ \Phi &= \ln \left[C^2 \left(\mathrm{tanh}t \right)^{-\sqrt{3}} + D^2 \left(\mathrm{tanh}t \right)^{\sqrt{3}} \right] \\ \sigma &= \frac{AC \left(\mathrm{tanh}t \right)^{-\sqrt{3}} + BD \left(\mathrm{tanh}t \right)^{\sqrt{3}}}{C^2 \left(\mathrm{tanh}t \right)^{-\sqrt{3}} + D^2 \left(\mathrm{tanh}t \right)^{\sqrt{3}}}. \end{aligned} \tag{27}$$

This is the general solution for the negatively curved FRW string cosmology [3].

The Joseph vacuum solution was generalized by Ellis and MacCallum to Bianchi type VI_h [20]:

$$ds^2 = \sinh 2t \left[A^b \left(-dt^2 + dx^2 \right) + Ae^{2(1+b)x} dy^2 + A^{-1} e^{2(1-b)x} dz^2 \right], \tag{28}$$

where

$$A \equiv \left(\sinh 2t \right)^b \left(\mathrm{tanh}t \right)^{\sqrt{3+b^2}} \tag{29}$$

and $b^2 \equiv -1/h$. This corresponds to the Joseph type V solution when $b = 0$. The mirror image of the Ellis–MacCallum type VI_h cosmology is

$$ds^2 = \sinh 2t \left[A^b \left(-dt^2 + dx^2 \right) + e^{2x} \left(dy^2 + dz^2 \right) \right], \tag{30}$$

where the dilaton and axion fields are given by Eqs. (21) and (22), respectively, with

$$e^p = \left(\sinh 2t \right)^b \left(\mathrm{tanh}t \right)^{\sqrt{3+b^2}} e^{2bx}. \tag{31}$$

A calculation of the structure constants of the isometry group of Eq. (30) implies that it is a LRS Bianchi type V cosmology. Furthermore, the fluid flows associated with the dilaton and axion fields are not orthogonal to the surfaces of homogeneity, $t =$ constant. Thus, Eq. (30) represents a tilted LRS Bianchi type V string cosmology.

Finally, we consider the vacuum type VII_h plane wave solution [21]:

$$ds^2 = -dt^2 + s^{-2} t^2 dx^2 + t^{2s} e^{-2x} dl^2, \tag{32}$$

where

$$dl^2 = \cosh \mu \left(dy^2 + dz^2 \right) - \sinh \mu \left[\left(dy^2 - dz^2 \right) \cos 2ku + 2dydz \sin 2ku \right] \tag{33}$$

and $u \equiv x - s \ln t$, $s^{-1} \equiv 1 + k^2 \sinh^2 \mu$ and $\{k, \mu\}$ are constants. This metric may be interpreted as two monochromatic, circularly–polarized gravitational waves with vectors $\pm\mathbf{k}$ travelling in the $\pm x$ directions with constant amplitudes μ [22]. It can be shown that this metric admits a covariantly constant null Killing vector field, $l_\nu \equiv \partial_\nu u$, $l_\nu l^\nu = 0$, and is therefore an exact solution to the classical string equations of motion to *all* orders in the inverse string tension [23].

The mirror image of the Lukash solution is

$$ds^2 = -dUdV + U^{2s}\left(dy^2 + dz^2\right), \tag{34}$$

where $U \equiv te^{-x/s}$ and $V \equiv te^{x/s}$ and the dilaton and axion fields are given by

$$e^\Phi = \cosh\mu - \sinh\mu\cos 2ku \tag{35}$$

$$\sigma = -\frac{\sin 2ku \sinh\mu}{\cosh\mu - \sinh\mu\cos 2ku}, \tag{36}$$

respectively. The energy momentum tensor for the matter fields may be written in the form $T_{\mu\nu} = \Pi(u) l_\mu l_\nu$, where $\Pi = \Pi(u)$ is a scalar function of the light–cone coordinate u. Thus, Eq. (34) may be interpreted as a conformally flat plane wave background with pure radiation (null dust). It is interesting because it is also exact to all order in α' due to the existence of the covariantly constant null Killing vector field. The dilaton field oscillates with a constant frequency and is bounded from above and below. In the string frame, this field parametrizes the effective gravitational (string) coupling, $g_s^2 \equiv e^\Phi$, and Eq. (34) is therefore an example of an exact string solution with an oscillating, but bounded, gravitational 'constant'.

We conclude with a discussion of further applications of the symmetry (12). It is important to emphasize that different space–time interpretations apply to solutions related by Eq. (12). This is evident from the simple derivation of the general, negatively curved FRW string cosmology from the Jacobs vacuum type V solution. In general, Eq. (12) may be employed to derive inhomogeneous G_2 string cosmologies with non–trivial dilaton and axion fields from solutions where neither of these fields is initially trivial. This includes backgrounds constructed from conformal field theories (see Ref. [7] and references therein).

These string models have a number of important physical applications. They allow density perturbations in string–inspired inflationary models such as the pre–big bang scenario to be studied [25]. The propagation of gravitational waves in string backgrounds may also be analyzed in terms of G_2 space–times [26]. Furthermore, the collision of self–gravitating plane waves can be modelled as the time reversal of a G_2 cosmology in the vicinity of the big bang singularity [24].

We applied the discrete symmetry (12) within the context of vacuum general relativity. This is interesting because exact, vacuum G_2 solutions to Einstein gravity have been extensively studied [27] and a large number of solution–generating techniques are known [10]. New algorithms that include Eq. (12) may be developed to generate the corresponding string cosmologies. For example, there exist a number of methods for generating inequivalent G_2 vacuum solutions with off–diagonal terms in the spatial metric [28]. Together with Eq. (12), these will lead to different string solutions to those derived in this work. Alternatively, cosmological models with a single, minimally coupled scalar field may be generated from vacuum solutions by means of the algorithm introduced by Barrow [29] and developed by Wainwright, Marshman and Ince [30]. These solutions could then serve as seeds for generating string backgrounds in the manner outlined above. Moreover, once a solution with non–trivial dilaton and two–form potential has been derived in this way, more general type II string backgrounds with non–trivial Ramond–Ramond fields can be found directly [31].

Finally, we outline a related method for deriving four–dimensional string cosmologies from vacuum Einstein gravity. The SL$(2, R)$ symmetry (7) of the effective action (5) may be interpreted geometrically by considering the compactification of six–dimensional vacuum Einstein gravity, $S = \int d^6x\sqrt{-g_6}R_6$, on a non–dynamical two–torus. Assuming the *ansatz*

$$ds_6^2 = {}^{(4)}g_{\mu\nu}(x)dx^\mu dx^\nu + e^{-\Phi(x)}dy_6^2 + e^{\Phi(x)}\left(dy_5 + \sigma(x)dy_6\right)^2 \tag{37}$$

for the higher–dimensional metric and integrating over the spatial variables y^i ($i = 5, 6$) then leads to an effective four–dimensional action that is formally equivalent to Eq. (5) if the dilaton and axion fields are identified with the appropriate degrees of freedom in the metric (37).

Thus, four–dimensional string backgrounds may be derived directly from six–dimensional, Ricci–flat solutions. This geometrical interpretation is similar to the recently proposed conjecture that the SL$(2, Z)$ symmetry of the ten–dimensional type IIB superstring arises from the toroidal compactification of twelve–dimensional F–theory [32]. It is closely related to a theorem of differential geometry due to Campbell that states that a Riemannian manifold of dimension n may be locally and isometrically embedded in a Ricci–flat, Riemannian manifold of dimension $N \geq n+1$ [33]. It would be interesting to investigate this correspondence further to derive non–trivial string models. Applying Campbell's theorem to embed lower–dimensional manifolds in six–dimensions would lead to a wide class of Ricci–flat solutions of the form given by Eq. (37). The $\{y^i\}$ components of the embedding six–dimensional metric would then directly determine the behaviour of the dilaton and axion fields in four dimensions.

Acknowledgments

The author is supported by the Particle Physics and Astronomy Research Council (PPARC), UK. We thank J. D. Barrow, A. Coley, K. E. Kunze and D. Wands for helpful discussions.

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